Definition

Model

A mathematical model is a representation of a phenomena by means of mathematical equations. If the phenomena is growth, the corresponding model is called a growth model. Here we are going to study the following 3 models.

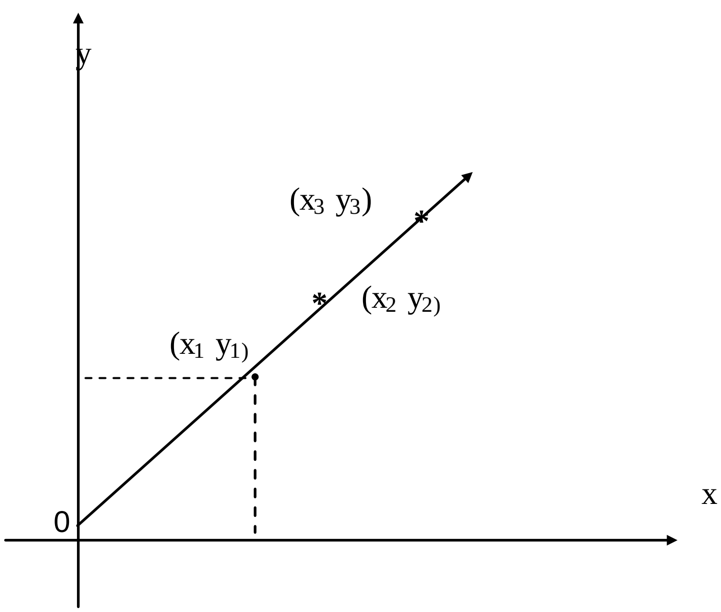
1. linear model
2. Exponential model
3. Power model

1. Linear model

The general form of a linear model is y = a+bx. Here both the variables x and y are of degree 1.

To fit a linear model of the form y=a+bx to the given data.

Here a and b are the parameters (or) constants of the model. Let (x_1, y_1) (x_2, y_2)………… (x_n, y_n) be n pairs of observations. By plotting these points on an ordinary graph sheet, we get a collection of dots which is called a scatter diagram.

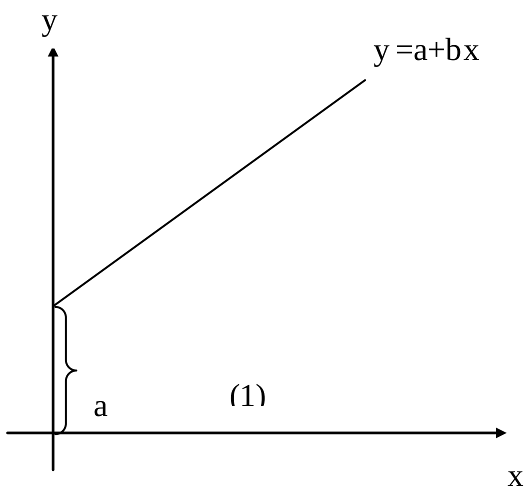


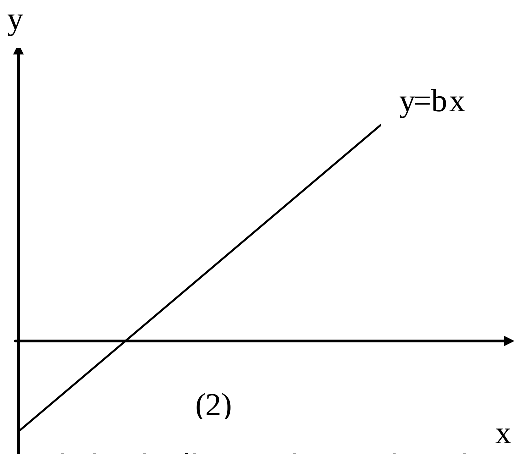
There are two types of linear models

(i) y = a+bx (with constant term)

(ii) y = bx (without constant term)

The graphs of the above models are given below :





‘a’ stands for the constant term which is the intercept made by the line on the y axis. Wh a=0 =0, y =a ie ‘a’ is the intercept, ‘b’ stands for the slope of the line .

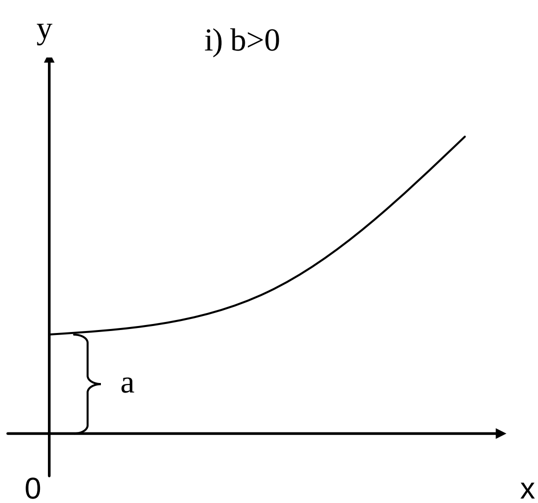
Eg:1. The table below gives the DMP(kgs) of a particular crop taken at different stages; fit a linear growth model of the form w=a+bt, and find the value of a and b from the graph.

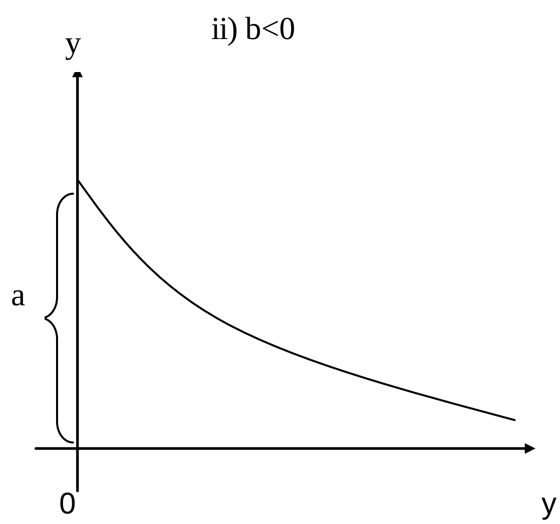
t (in days) ;	0	5	10	20	25
DMP w: (kg/ha)	2	5	8	14	17

2. Exponential model

This model is of the form $y = ae^{bx}$ where a and b are constants to be determined

The graph of an exponential model is given below.





Note:

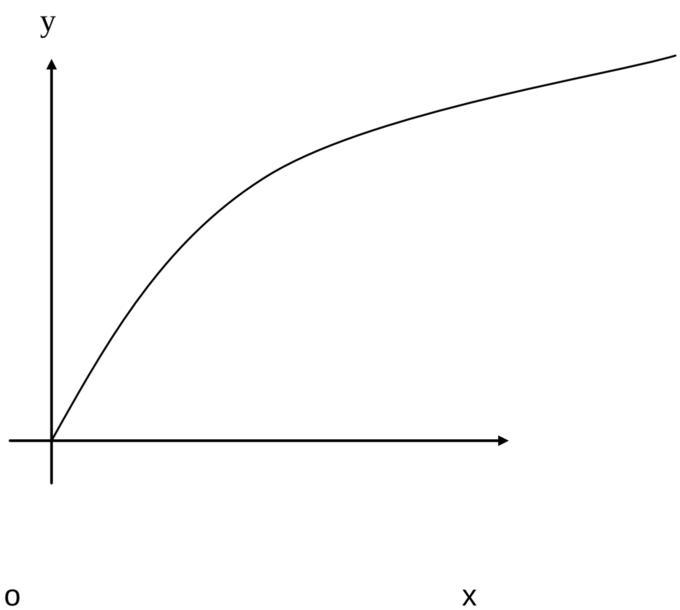
The above model is also known as a semilog model. When the values of x and y are plotted on a semilog graph sheet we will get a straight line. On the other hand if we plot the points xand y on an ordinary graph sheet we will get an exponential curve.

Eg: 2. Fit an exponential model to the following data.

x in days	5	15	25	35	45
y in mg per plant	0.05	0.4	2.97	21.93	162.06

Power Model

The most general form of the power model is $y = ax^b$



Example: Fit the power function for the following data

x	0	1	2	3
y	0	2	16	54

Crop Response models

The most commonly used crop response models are

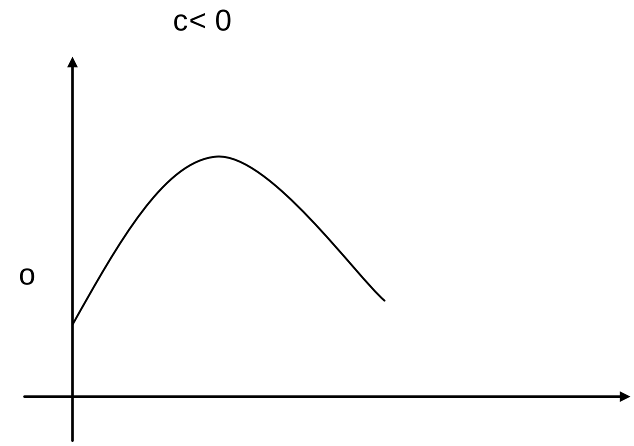
i) Quadratic model

ii) Square root model

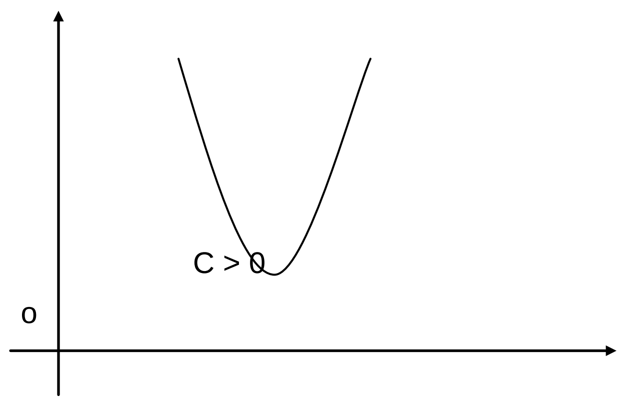
Quadratic model

The general form of quadratic model is $y = a + b x + c x^2$

When c< 0 the curve attains maximum at its peak.



When c > 0 the curve attains minimum at its peak.



The parabolic curve bends very sharply at the maximum or minimum points.

Example

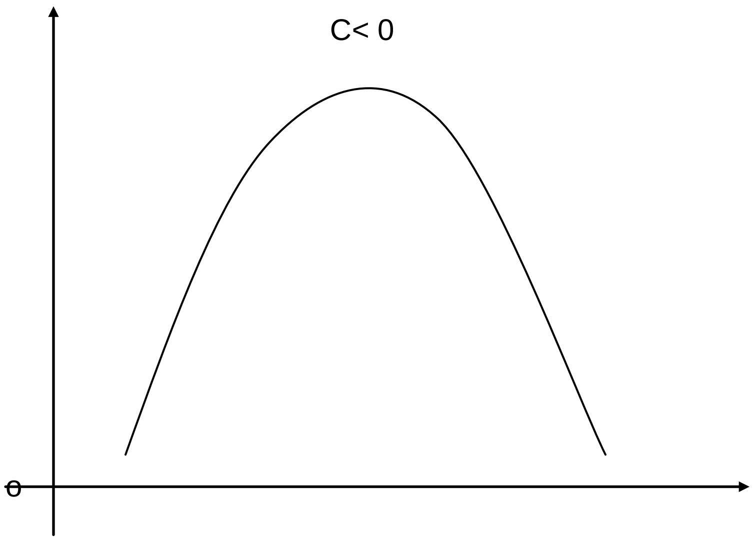
Draw a curve of the form y = a + b x + c x^2 using the following values of x and y

x	0	1	2	4	5	6
y	3	4	3	-5	-12	-21

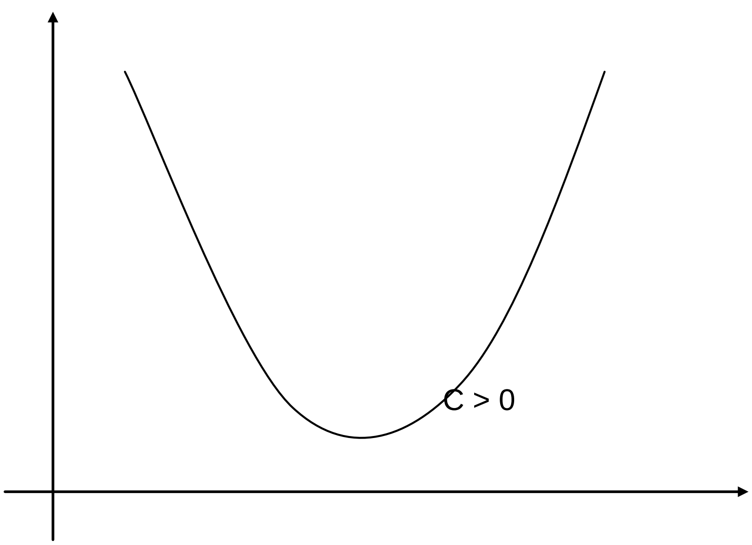
Square root model

The standard form of the square root model is $y = a + b\sqrt{x} + cx$

When c is negative the curve attains maximum



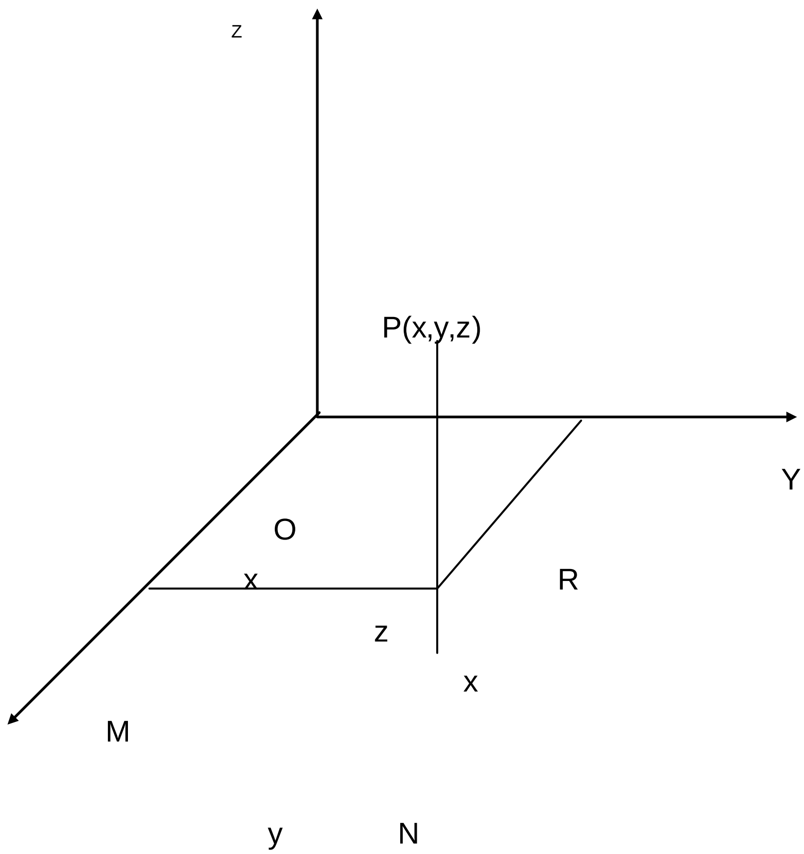
The curve attains minimum when c is positive.



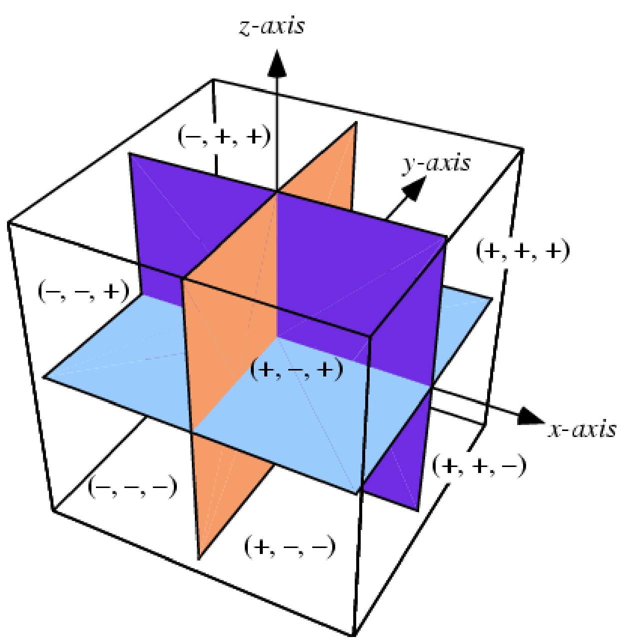
At the extreme points the curve bends at slower rate

Three dimensional Analytical geometry

Let OX ,OY & OZ be mutually perpendicular straight lines meeting at a point O. The extension of these lines OX^1, OY^1 and OZ^1 divide the space at O into octants(eight). Here mutually perpendicular lines are called X, Y and Z co-ordinates axes and O is the origin. The point P (x, y, z) lies in space where x, y and z are called x, y and z coordinates respectively.



where NR = x coordinate, MN = y coordinate and PN = z coordinate



Distance between two points

The distance between two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

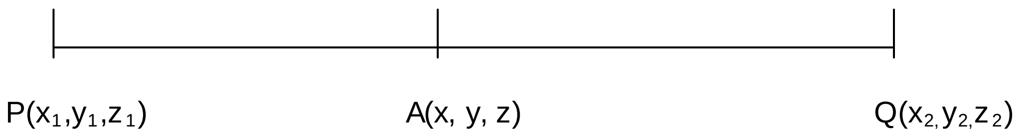
dist AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

In particular the distance between the origin O (0,0,0) and a point P(x,y,z) is

OP = $\sqrt{x^2 + y^2 + z^2}$

The internal and External section

Suppose $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are two points in three dimensions.

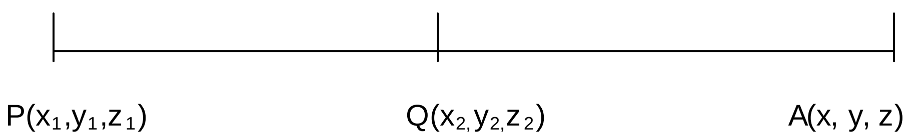


The point A(x, y, z) that divides distance PQ internally in the ratio $m_1 : m_2$ is given by

$$A = \left[\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right]$$

Similarly

$P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are two points in three dimensions.



The point A(x, y, z) that divides distance PQ externally in the ratio $m_1 : m_2$ is given by

$$A = \left[\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2}\right]$$

If A(x, y, z) is the midpoint then the ratio is 1:1

$$A = \left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right]$$

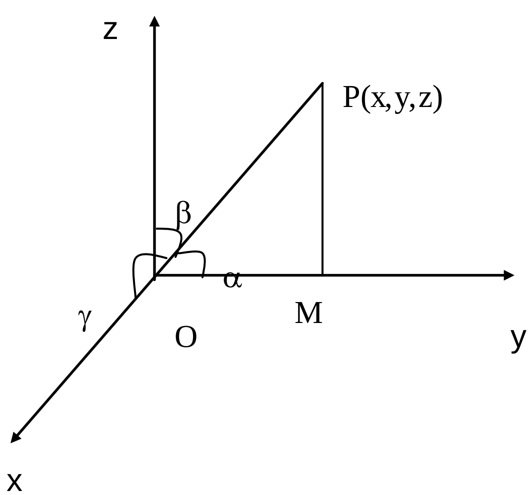
Problem

Find the distance between the points P(1,2-1) & Q(3,2,1)

PQ= $\sqrt{(3-1)^2 + (2-2)^2 + (1+1)^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

Direction Cosines

Let P(x, y, z) be any point and OP = r. Let α,β,γ be the angle made by line OP with OX, OY & OZ. Then α,β,γ are called the direction angles of the line OP. cos α, cos β, cos γ are called the direction cosines (or dc's) of the line OP and are denoted by the symbols l, m ,n.



Result

By projecting OP on OY, PM is perpendicular to y axis and the $\angle POM = \beta$ also OM = y

$$\therefore \cos\beta = \frac{y}{r}$$

Similarly, $\cos\alpha = \frac{x}{r}$

$$\cos\gamma = \frac{z}{r}$$

(i.e) $l = \frac{x}{r}$, $m = \frac{y}{r}$, $n = \frac{z}{r}$

$$\therefore l^2 + m^2 + n^2 = \frac{x^2 + y^2 + z^2}{r^2}$$

$(\because r = \sqrt{x^2 + y^2 + z^2} \Rightarrow$ Distance from the origin)

$$\therefore l^2 + m^2 + n^2 = \frac{x^2 + y^2 + z^2}{x^2 + y^2 + z^2} = 1$$

$l^2 + m^2 + n^2 = 1$

(or) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

Note

The direction cosines of the x axis are (1,0,0)

The direction cosines of the y axis are (0,1,0)

The direction cosines of the z axis are (0,0,1)

Direction ratios

Any quantities, which are proportional to the direction cosines of a line, are called direction ratios of that line. Direction ratios are denoted by a, b, c.

If l, m, n are direction cosines an a, b, c are direction ratios then

$a \propto l$, $b \propto m$, $c \propto n$

(ie) $a = kl$, $b = km$, $c = kn$

(ie) $\frac{a}{l} = \frac{b}{m} = \frac{c}{n} = k$ (Constant)

(or) $\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{1}{k}$ (Constant)

To find direction cosines if direction ratios are given

If a, b, c are the direction ratios then direction cosines are

$$\left.\begin{aligned} \frac{l}{a} = \frac{1}{k} \Rightarrow \quad l &= \frac{a}{k} \\ \text{similarly} \quad m &= \frac{b}{k} \\ n &= \frac{c}{k} \end{aligned}\right\} \quad (1)$$

$l^2+m^2+n^2 = \frac{1}{k^2}(a^2+b^2+c^2)$

(ie) $1 = \frac{1}{k^2}(a^2+b^2+c^2)$

$\Rightarrow k^2 = a^2+b^2+c^2$

Taking square root on both sides

K = $\sqrt{a^2+b^2+c^2}$

$\therefore l = \frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \frac{c}{\sqrt{a^2+b^2+c^2}}$

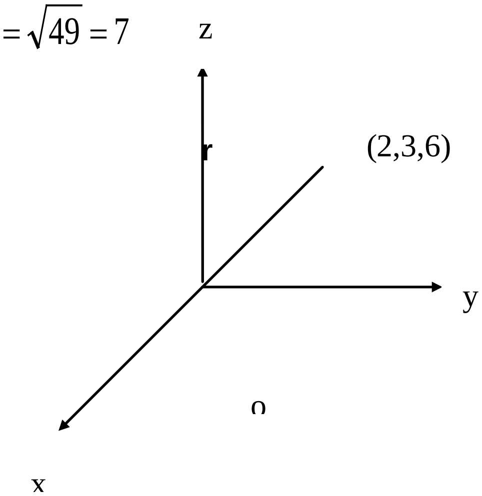
Problem

1. Find the direction cosines of the line joining the point (2,3,6) & the origin.

Solution

By the distance formula

$r = \sqrt{x^2+y^2+z^2} = \sqrt{2^2+3^2+6^2} = \sqrt{4+9+36} = \sqrt{49} = 7$



Direction Cosines are

***l* = cos** $\propto$ **=** $\frac{x}{r} = \frac{2}{7}$

***m* = cos** β **=** $\frac{y}{r} = \frac{3}{7}$

***n* = cos** γ **=** $\frac{z}{r} = \frac{6}{7}$

2. Direction ratios of a line are 3,4,12. Find direction cosines

Solution

Direction ratios are 3,4,12

(ie) a = 3, b = 4, c = 12

Direction cosines are

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{3}{\sqrt{3^2+4^2+12^2}} = \frac{3}{\sqrt{169}} = \frac{3}{13}$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{4}{\sqrt{3^2+4^2+12^2}} = \frac{4}{\sqrt{169}} = \frac{4}{13}$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{12}{\sqrt{3^2+4^2+12^2}} = \frac{12}{\sqrt{169}} = \frac{12}{13}$$

Note

1) The direction ratios of the line joining the two points A(x_1, y_1, z_1) & B (x_2, y_2, z_2) are ($x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$)

2) The direction cosines of the line joining two points A (x_1, y_1, z_1) & B (x_2, y_2, z_2) are $\frac{x_2 - x_1}{r}, \frac{y_2 - y_1}{r}, \frac{z_2 - z_1}{r}$

r = distance between AB.